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QED and string theory

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Abstract: We analyze the D9-$\overline{\text{D9}}$ system in type IIB string theory using Dp-brane probes. It is shown that the world-volume theory of the probe Dp-brane contains two-dimensional and four-dimensional QED in the cases with $p = 1$ and $p = 3$, respectively, and some applications of the realization of these well-studied quantum field theories are discussed. In particular, the two-dimensional QED (the Schwinger model) is known to be a solvable theory and we can apply the powerful field theoretical techniques, such as bosonization, to study the D-brane dynamics. The tachyon field created by the D9-$\overline{\text{D9}}$ strings appears as the fermion mass term in the Schwinger model and the tachyon condensation is analyzed by using the bosonized description. In the T-dualized picture, we obtain the potential between a D0-brane and a D8-$\overline{\text{D8}}$ pair using the Schwinger model and we observe that it consists of the energy carried by fundamental strings created by the Hanany-Witten effect and the vacuum energy due to a cylinder diagram. The D0-brane is treated quantum mechanically as a particle trapped in the potential, which turns out to be a system of a harmonic oscillator.
As another application, we obtain a matrix theory description of QED using Taylor's T-duality prescription, which is actually applicable to a wide variety of field theories including the realistic QCD. We show that the lattice gauge theory is naturally obtained by regularizing the matrix size to be finite.

Keywords: D-branes, Brane Dynamics in Gauge Theories, Tachyon Condensation.

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Contents

1. Introduction

The interplay between string theory and quantum field theory has been one of the most successful subject during the second revolution of string theory. There are many things that we can learn from it. For example, quantum field theory often provides useful tools to study non-perturbative aspects of string theory. Even though the non-perturbative formulation of string theory is not available yet, we can analyze non-perturbative effects using the techniques developed in the quantum field theory once we know the realization of the quantum field theory in string theory. On the other hand, we can apply various string duality (such as S-duality, T-duality, M-theory, open/closed duality, etc.) to quantum field theory. If we are lucky enough, we would be able to obtain a new description of the quantum field theory. However, most of the works along this line is done in supersymmetric situations. Since our goal is to understand the real world, it would be quite important to investigate non-supersymmetric models.

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One of the purpose of this paper is to analyze unstable D-brane systems[1] by using probe D-branes. As a typical example, we consider the D9-$\overline{\mathrm{D9}}$ system in type IIB string theory and take a Dp-brane ($p = 1, 3$) as a probe [2]. As we will explain in section 2, the world-volume theory on the Dp-brane contains $(p+1)$ dimensional QED.

The realization of the four dimensional QED in string theory could be interesting since it is a realistic system. It would be interesting if we could say something realistic using string theory, though we will not consider much about it in this paper.

In the $p = 1$ case, we obtain the two dimensional QED, which is often called as the Schwinger model. The Schwinger model is known to be one of the exactly solvable interacting quantum field theories [3]. Actually, it has been shown that the system is equivalent to a free massive scalar field theory by using bosonization techniques [4]. We can thus analyze the D-brane dynamics using the field theoretical results in the Schwinger model. Being two-dimensional, it is not a realistic model, however there are many features common to the four dimensional QCD (such as confinement, chiral symmetry breaking, axial anomaly, instantons, θ-vacuum etc.) which make this theory even more interesting.

The D9-$\overline{\mathrm{D9}}$ system is an unstable D-brane system and it is known that there is a tachyon field created by the open string stretched between the D9-brane and the $\overline{\mathrm{D9}}$-brane. When the tachyon field homogeneously condenses, the D9-$\overline{\mathrm{D9}}$ pairs are believed to be annihilated [5, 6, 1]. More interestingly, when the tachyon field takes a vortex configuration, it represents a D7-brane [7, 1]. The lower dimensional D-branes can be similarly constructed by non-trivial tachyon configurations in the D9-$\overline{\mathrm{D9}}$ system [8]. When we put the D-brane probe in this system, the tachyon field is interpreted as the fermion mass parameter in the world-volume theory. We will observe these phenomena in terms of the bosonized description of the Schwinger model.

When we compactify the direction parallel to the D-brane probe to a torus, we can T-dualize the system to obtain a lower dimensional description of QED. The T-duality prescription given in [9]–[12] can also be applied to the world-volume theory on the D-brane in the presence of the D9-$\overline{\mathrm{D9}}$ pairs. Actually, since the essential step in the prescription is just the Fourier transformation, it is applicable to any field theory compactified on a torus. By T-dualizing all the space-time directions, we obtain a matrix theory description. The size of the matrix variables here is infinite, since there are infinitely many copies of the D-branes in the covering space. We shall show that when we regularize the matrix size to be finite in this matrix theory description, we naturally obtain usual lattice gauge theory.

The paper is organized as follows. First we summarize the world-volume theory of the Dp-brane probe in the D9-$\overline{\mathrm{D9}}$ system and see how QED is realized on it in section 2. In section 3, we study D-brane dynamics using the Schwinger model. After a brief review of the bosonization, we consider tachyon condensation in terms of the bosonized description of the Schwinger model. In section 4, we T-dualize the spatial direction of the D1-brane probe and consider a D0-brane in the presence of a D8-$\overline{\mathrm{D8}}$ pair. T-duality is more systematically considered in section 5, in which the relation between the matrix regularization and the

[1]See [1] for a review.

lattice theory is discussed. Section 6 is devoted to discussion and we make some speculation on the S-duality of the D9-$\overline{\mathrm{D9}}$ system, which was actually our first motivation for the present work.

2. QED in string theory

In this section, we consider the D9-$\overline{\mathrm{D9}}$ system in type IIB string theory and take a Dp-brane (p: odd) as a probe.[2] We summarize the world-volume theory of the Dp-brane probe fixing our notation. We will soon show that the world-volume theory of the probe Dp-brane contains two-dimensional and four-dimensional QED in the cases with $p = 1$ and $p = 3$, respectively.

2.1 Dp-branes in the type IIB D9-$\overline{\mathrm{D9}}$ system

Let us consider n Dp-branes extended along $x^0, \ldots, x^p$ directions in the presence of background N D9-brane - $\overline{\mathrm{D9}}$-brane pairs. The world-volume fields on the Dp-branes consist of those created by the p-p strings, p-9 strings and p-$\overline{9}$ strings, which are the open strings with ends on the respective D-branes.

The massless fields generated by the p-p strings are the same as those obtained by the dimensional reduction of 10 dimensional $U(n)$ super Yang-Mills theory, namely, a gauge field A_μ ($\mu = 0, \ldots, p$), scalar fields Φ^i ($i = p+1, \cdots, 9$) and fermions S. These fields transform as the adjoint representation of the gauge group $\mathrm{U}(n)$.

As for the p-9 strings and p-$\overline{9}$ strings, the mass of the lowest modes depends on p. In fact, in the mass shell condition $L_0 \,|\, \mathrm{phys}\,\rangle = a \,|\, \mathrm{phys}\,\rangle$, the zero-point energy a is given by

$$a^{\mathrm{R}} = 0\,, \qquad a^{\mathrm{NS}} = (p-1)\left(-\frac{1}{24} - \frac{1}{48}\right) + (9-p)\left(\frac{1}{48} + \frac{1}{24}\right) = \frac{5-p}{8} \tag{2.1}$$

for R-sector and NS-sector, respectively [13]. As we can see from (2.1), the lowest mass states in the NS-sector are massive for $p = 1, 3$, massless for $p = 5$ and tachyonic for $p = 7$. From now, we shall concentrate on the $p = 1, 3$ cases, in which we can simply forget about the extra massless or tachyonic bosons coming from the p-9 and p-$\overline{9}$ strings in the low energy physics.

On the other hand, since the normal ordering constant for the R-sector a^R is always zero, we have massless fermions. Note that the world-sheet fermions ψ^i ($i = p+1, \ldots, 9$) which correspond to the directions transverse to the Dp-brane do not have zero modes, and hence the massless states are generated by the zero modes of ψ^μ ($\mu = 0, \ldots, p$). Then, the ground states in the R-sector consist of positive and negative chirality spinor representations of the Lorentz group $\mathrm{SO}(1, p)$. One of these two spinors is removed by GSO projection. We choose the positive chirality spinor as physical states created by the p-9 strings and the corresponding fields are denoted as λ_+^I, where $I = 1, \ldots, N$ labels the Chan-Paton indices of the D9-branes. These fermion fields are considered as complex

[2]This configuration is first analyzed by K. Hori in [2].

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field	U(n)	SO(1,1)	SO(8)	U(N) × U(N)
A_μ	**adj.**	2	1	$(1,1)$
Φ^i	**adj.**	1	8	$(1,1)$
S_+	**adj.**	1_+	8_+	$(1,1)$
S_-	**adj.**	1_-	8_-	$(1,1)$
λ_+^I	**fund.**	1_+	1	(**fund.**, 1)
$\lambda_-^{\bar{I}}$	**fund.**	1_-	1	(1, **fund.**)

Table 1: The massless fields on the D1-brane in the presence of D9-$\overline{\text{D9}}$ pairs. Here 1_+ and 1_- denote the positive and negative chirality Majorana-Weyl spinor representation of the Lorentz group SO(1,1), respectively. Similarly, 8_+ and 8_- denote the positive and negative chirality Majorana-Weyl spinor representation of the SO(8), respectively. **adj.** and **fund.** represent the adjoint and fundamental representations, respectively.

field	U(n)	SO(1,3)	SO(6)	U(N) × U(N)
A_μ	**adj.**	4	1	$(1,1)$
Φ^i	**adj.**	1	6	$(1,1)$
S	**adj.**	2_+	4_+	$(1,1)$
λ_+^I	**fund.**	2_+	1	(**fund.**, 1)
$\lambda_-^{\bar{I}}$	**fund.**	2_-	1	(1, **fund.**)

Table 2: The massless fields on the D3-brane in the presence of D9-$\overline{\text{D9}}$ pairs. Here 2_+ and 2_- denote the positive and negative chirality spinor representation of the Lorentz group SO(1,3). Similarly, 4_+ denotes the positive chirality spinor representation of the SO(6).

fermions, since the p-9 string have two orientations. The massless fermion fields generated by the p-$\overline{9}$ strings can be obtained in a similar way. The only point we should notice is that the GSO projection for the p-$\overline{9}$ strings is opposite to the one we chose for the p-9 strings. Therefore, the massless fermion fields coming from the p-$\overline{9}$ strings belong to the negative chirality spinor representation of the Lorentz group SO$(1,p)$ and these fields are denoted by $\lambda_-^{\bar{I}}$. Here $\bar{I} = 1,\ldots,N$ labels the Chan-Paton indices of the $\overline{\text{D9}}$-branes.

In summary, the massless fields on the Dp-branes for $p = 1,3$ are as listed in table 1 and table 2, respectively. The U(N) × U(N) symmetry on the tables 1 and 2 is the gauge symmetry of the N D9-$\overline{\text{D9}}$ pairs which can be seen as the global symmetry on the Dp-brane world-volume. Note that the anomaly with respect to the gauge symmetry U(n) on the world-volume precisely cancels if and only if the number of the D9-branes and the $\overline{\text{D9}}$-branes are the same. This condition is also consistently required from the cancellation of the RR-tadpole or the 10 dimensional gauge anomaly in the D9-$\overline{\text{D9}}$ system [14, 15]. It is sometimes useful to combine λ_+ and λ_- to make Dirac fermions

$$\lambda^I = \begin{pmatrix} \lambda_+^I \\ \lambda_-^I \end{pmatrix}, \qquad (I = 1,\ldots,N)\,, \tag{2.2}$$

though only the diagonal U(N) component of the U(N)×U(N) symmetry becomes manifest in this notation.

When $n = 1$, the world-volume theory of the Dp-brane becomes very simple, since the gauge group U(n) is abelian. Then, the low energy world-volume action is

$$S_{\mathrm{D}p} = \int d^{p+1}x \left\{ -\frac{1}{4g_{\mathrm{YM}}^2} F_{\mu\nu}F^{\mu\nu} + i\overline{\lambda}_I \gamma^\mu D_\mu \lambda^I + \frac{1}{2}\partial_\mu \Phi^i \partial^\mu \Phi^i + i\overline{S}\gamma^\mu \partial_\mu S \right\}, \tag{2.3}$$

where $D_\mu = \partial_\mu + iA_\mu$ is the covariant derivative. The gauge coupling g_{YM} is related to the string coupling g_s and the string length l_s by

$$g_{\mathrm{YM}}^2 = \frac{g_s}{l_s}(2\pi l_s)^{p-2}\,. \tag{2.4}$$

We take the limit $l_s \to 0$ keeping g_{YM} fixed, in which higher order terms in the action (2.3) as well as the coupling to the bulk fields are suppressed. In the action (2.3), the U(1) gauge field and the Dirac fermion λ^I make QED with N flavors, while the fields Φ^i and S are decoupled from this sector.

2.2 Turning on bulk fields

It is also interesting to see what happens if we turn on the bulk fields as a background. When we turn on the R-R fields, we should take into account the Chern-Simons term

$$S_{CS} = \mu_p \int_{p+1} C \wedge \mathrm{tr}\left(e^{2\pi\alpha' F}\right), \tag{2.5}$$

where $C = C_0 + C_2 + \ldots$ is a formal sum of the R-R k-form fields C_k [16]–[18]. In particular, the R-R 0-form field C_0 acts as a theta parameter in the Dp-brane world-volume theory. For example, in the case with $p = 1$, we have

$$S_{C_0} = \int C_0\, F\,, \tag{2.6}$$

which will play an important role in the following sections.

The tachyon field T created by 9-$\overline{9}$ strings will couple to the fermions λ^I as

$$S_T = \int d^{p+1}x\ T^I{}_{\bar{I}}\, \lambda^*_{+I}\lambda^{\bar{I}}_- + \mathrm{h.c.}\,, \tag{2.7}$$

which behaves as a mass term for the fermions when the tachyon field is non-zero.[3] As observed in [2], when the tachyon get large vev, the fermions λ^I will become massive and decouple from the low energy degrees of freedom, which is consistent with the annihilation of the D9-$\overline{\mathrm{D9}}$ pairs via the tachyon condensation [5, 6, 1].

When we turn on the bulk gauge fields $\mathcal{A}_M$ and $\overline{\mathcal{A}}_M$ which correspond to the U(N) × U(N) gauge symmetry of the D9-$\overline{\mathrm{D9}}$ system, we should add

$$S_{\mathcal{A}} = -\int d^{p+1}x \overline{\lambda}_I \gamma^\mu (\mathcal{A}^+_\mu + \gamma_5 \mathcal{A}^-_\mu)^I{}_J \lambda^J, \tag{2.8}$$

[3]Here we assume that the possible T dependence in the kinetic term of the fermion λ^I can be absorbed by the redefinition of the fermion fields. More general T dependence in (2.7) such as $S_T = \int d^{p+1}x\, f(T)^I{}_{\bar{I}}\lambda^*_{+I}\lambda^{\bar{I}}_- +$h.c. is also assumed to be absorbed by the redefinition of the tachyon field $f(T) \to T$. The T dependence of the kinetic term of the gauge field can appear as the higher loop corrections which will vanish in the decoupling limit for $p = 1$.

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where $\mathcal{A}_\mu^\pm = \frac{1}{2}(\mathcal{A}_\mu + \mathcal{A}_i \partial_\mu X^i \pm (\overline{\mathcal{A}}_\mu + \overline{\mathcal{A}}_i \partial_\mu X^i))$, ($X^i \equiv T_p^{-1/2}\Phi^i$ where T_p is the Dp-brane tension). Therefore $\mathcal{A}_\mu^+$ and $\mathcal{A}_\mu^-$ can be regarded as sources which couple to the vector and axial currents $j^\mu = \overline{\lambda}\gamma^\mu\lambda$ and $j_5^\mu = \overline{\lambda}\gamma^\mu\gamma_5\lambda$, respectively.

3. QED$_2$ and D-brane dynamics

In this section, we consider the $p = 1$ case in which we can realize the two-dimensional QED as we have seen in the last section. The two-dimensional massless QED (the massless Schwinger model) is one of the exactly solvable interacting quantum field theories [3]. It has been shown that the system is equivalent to a free massive scalar field theory by using bosonization techniques [4, 19, 20]. Here we will discuss various field theoretical results in the two-dimensional QED in terms of string theory and consider some applications to the D-brane dynamics.

3.1 The Schwinger model and bosonization

Here we briefly review some of the results in the Schwinger model.[4] We restrict our discussion to the one flavor case in this subsection for simplicity. The multi-flavor case will be discussed in section 3.5.

The action of the (massless) Schwinger model is

$$S = \int d^2x \left\{ -\frac{1}{4g_{\mathrm{YM}}^2} F^{\mu\nu}F_{\mu\nu} + i\overline{\lambda}\gamma^\mu(\partial_\mu + iA_\mu)\lambda \right\}, \tag{3.1}$$

where $\lambda = (\lambda_+, \lambda_-)^T$ is a complex Dirac fermion. We take the representation of the gamma matrices as $\gamma^0 = \sigma^1$, $\gamma^1 = -i\sigma^2$ and $\gamma^5 = \gamma^0\gamma^1 = \sigma^3$, where σ^i are the Pauli matrices.

The action (3.1) is invariant under the vector-like $\mathrm{U}(1)_V$ and axial $\mathrm{U}(1)_A$ transformations

$$\mathrm{U}(1)_V : \lambda \to e^{i\alpha}\lambda\,, \tag{3.2}$$

$$\mathrm{U}(1)_A : \lambda \to e^{i\alpha\gamma_5}\lambda\,, \tag{3.3}$$

though the later is anomalous. Correspondingly, we have the vector current $j^\mu = \overline{\lambda}\gamma^\mu\lambda$ and the axial current $j_5^\mu = \overline{\lambda}\gamma^\mu\gamma^5\lambda$ which satisfy

$$\partial_\mu j^\mu = 0\,, \qquad \partial_\mu j_5^\mu = \frac{1}{\pi}F_{01}\,. \tag{3.4}$$

Remarkably, using the bosonization techniques, it has been shown that the Schwinger model is equivalent to a two-dimensional system of a free massive scalar field with a standard hamiltonian

$$H = \frac{1}{2}\int dx^1 \left\{ \pi_\varphi^2 + (\partial_1\varphi)^2 + m^2\varphi^2 \right\}, \tag{3.5}$$

[4]See for example [19, 20] for a review of the Schwinger model and its bosonization.

where π_φ is the momentum conjugate to the scalar field φ and the mass is related to the gauge coupling by $m^2 = g_{\rm YM}^2/\pi$. The scalar field φ and its conjugate momentum π_φ is related to the field strength and axial charge density, respectively, as

$$\varphi = -\frac{\sqrt{\pi}}{g_{\rm YM}^2} F_{01}\,, \qquad \pi_\varphi = \sqrt{\pi}\,\lambda^\dagger \sigma^3 \lambda\,. \tag{3.6}$$

Using the equation of motion

$$\partial_\mu F^{\mu\nu} = g_{\rm YM}^2 j^\nu \tag{3.7}$$

these relations can be written in a covariant form as

$$j_5^\mu = \epsilon^{\mu\nu} j_\nu = \frac{1}{\sqrt{\pi}} \partial^\mu \varphi\,, \tag{3.8}$$

where $\epsilon^{10} = -\epsilon^{01} = +1$. It is easy to see that the equation (3.4) is consistent with the equation of motion for the free scalar field φ under this correspondence (3.8).

3.2 Axial anomaly and the Green-Schwarz mechanism

As we mentioned in section 3.1, the axial $\mathrm{U}(1)_A$ symmetry is anomalous in the Schwinger model. This fact might sound puzzling since this axial $\mathrm{U}(1)_A$ symmetry is a part of the $\mathrm{U}(1) \times \mathrm{U}(1)$ gauge symmetry of the D9-$\overline{\rm D9}$ system which has to be anomaly free as a consistent theory. The resolution of this puzzle is obtained by a standard anomaly inflow argument as given in [18] which we shortly explain in the following.

The anomaly cancellation in the type IIB D9-$\overline{\rm D9}$ system was first discussed in [14] and further developed in [15]. It was shown that the anomaly is canceled by the Green-Schwarz mechanism [21] which requires the non-trivial gauge transformation rules for the R-R fields. In particular, the R-R 0-form field C_0 transforms as

$$C_0 \to C_0 + \frac{\alpha}{\pi} \tag{3.9}$$

under the axial $\mathrm{U}(1)_A$ transformation (3.3). Accordingly, the Chern-Simons term (2.6) will be shifted by $\frac{\alpha}{\pi}\int F$ which precisely cancels the anomalous transformation of the path integral measure of the fermion λ;

$$\mathcal{D}\lambda \to \mathcal{D}\lambda \,\exp\left(-i\,\frac{\alpha}{\pi}\int F\right). \tag{3.10}$$

Therefore there is actually no anomaly for the axial $\mathrm{U}(1)_A$ transformation when we combine (3.3) with the shift of the R-R 0-form field (3.9).

3.3 Massive Schwinger model and tachyon condensation

When the tachyon field becomes non-zero, the mass term for the fermion λ is induced as we saw in section 2.2. The two dimensional QED with a fermion mass term (the massive Schwinger model) is no longer a solvable theory, but we can still use the bosonization techniques. Actually, using the bosonization rule

$$\lambda_+^* \lambda_- \propto e^{i(2\sqrt{\pi}\varphi - \theta)}\,, \tag{3.11}$$

the bosonized action for the massive Schwinger model is obtained as [22, 23, 19, 20]

$$S=\int d^2x\left\{\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi-\frac{m^2}{2}\varphi^2+cT\left(\cos\left(2\sqrt{\pi}\,\varphi-\theta\right)-1\right)\right\},\qquad(3.12)$$

where $m^2=g_{\rm YM}^2/\pi=g_s/(2\pi^2\alpha')$, $\theta=2\pi C_0$ is the theta parameter, c is a numerical constant and T is the tachyon field appeared in (2.7), which is set to be real and non-negative by the axial $\mathrm{U}(1)_A$ transformation. Here we have added a constant term to the potential so that there is a finite energy stable vacuum in the $T\to\infty$ limit, since the D1-brane remains stable when the D9-$\overline{\rm D9}$ pair is annihilated via the tachyon condensation.

Let us now consider the tachyon condensation using this bosonized action (3.12). When the tachyon approach the minimum of the potential, we expect that T becomes very large. As we take the limit $T\to\infty$, the finite energy configurations are those with

$$\varphi=\sqrt{\pi}\left(n+\frac{\theta}{2\pi}\right),\qquad(n\in\mathbb{Z}),\qquad(3.13)$$

which means that the scalar field φ can no longer fluctuate.

The physical interpretation of the discrete values in (3.13) is clear from the relation (3.6). Since the scalar field φ is proportional to the electric flux F_{01} which induces the fundamental string charge, it represents that the D1-brane makes a bound state with n fundamental strings, which is called the $(n,1)$ string.

Then, the energy density $\mathcal{E}$ carried by the configuration (3.13) is

$$\mathcal{E}=\frac{m^2}{2}\pi\left(n+\frac{\theta}{2\pi}\right)^2=\frac{g_s}{4\pi\alpha'}(n+C_0)^2\,.\qquad(3.14)$$

This result can be reproduced by using the tension formula for the $(n,1)$ string. In fact, the tension of the $(n,1)$ string is given as [24, 25]

$$T_{(n,1)}=\frac{1}{2\pi\alpha'}\sqrt{(n+C_0)^2+\frac{1}{g_s^2}}\,,\qquad(3.15)$$

and hence the excess energy density of the system is

$$T_{(n,1)}-T_{\rm D1}=\frac{1}{2\pi\alpha' g_s}\left(\sqrt{1+g_s^2(n+C_0)^2}-1\right)\to\frac{g_s}{4\pi\alpha'}(n+C_0)^2\,,\qquad(3.16)$$

which agrees with (3.14) in the limit $g_s\to 0$ with $g_{\rm YM}^2=g_s/(2\pi\alpha')$ fixed.

It is interesting to note that we have obtained the correct quantization condition for the electric flux using our bosonized description of the D1-brane. The quantization of the electric flux is rather non-trivial if we use the DBI action to describe the system. (See [25, 12]) The action (3.12) also tells us that when T is finite, which corresponds to the situation that the tachyon field is not at the minimum of the potential, the scalar field φ can fluctuate and hence the electric flux is not fixed to the discrete values. Namely, the n-units of the fundamental string in the $(n,1)$ string is dissolved in the bulk. This is possible because the fundamental strings are unstable in the background of D9-$\overline{\rm D9}$ pairs, since they can decay to short pieces of open strings ending at the D9 or $\overline{\rm D9}$-branes.

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3.4 Kinks and vortices

Since the fermion mass is given by the tachyon field in our set up, it can vary along the D1-brane world-sheet. As an example, let us consider a kink-like tachyon configuration such as

$$T \sim x^1 , \tag{3.17}$$

where x^1 is the spatial direction along the D1-brane. In this case, since the tachyon field becomes large at $x^1 \to \pm\infty$, the scalar field φ takes discrete values as (3.13) at the spatial infinity. An interesting point here is that we can consider a kink configuration for the scalar field φ. Namely, the integer n in (3.13) chosen for $x^1 \to -\infty$ can be different from that for $x^1 \to +\infty$. This means that the fundamental string ingredients can escape from the D1-brane at the region where $T \sim 0$. The physical interpretation of this configuration is clear. The kinky tachyon profile (3.17) corresponds to a (non-BPS) D8-brane localized at $x^1 \sim 0$ [1, 7] and [26]–[29]. Since a fundamental string can end at the D8-brane, an $(n, 1)$ string can be changed to an $(n', 1)$ string with some integer n' different from n as the string crosses the D8-brane. This D8-brane is unstable and it will disappear when the tachyon on it condenses. In this case, the world-sheet theory on the D1-brane probe describes a string junction as in [30]. It is also possible to make a stable configuration with varying tachyon. For example, the vortex configuration

$$T = u(x^1 + ix^2) , \qquad (u \to \infty) \tag{3.18}$$

for the D9-$\overline{\text{D9}}$ tachyon is known to represent a BPS D7-brane [7, 8, 1]. The same argument as above can be applied to this configuration when we place the D1-brane at $x^2 = 0$.

It is also interesting to consider a tachyon vortex parallel to the D1-brane, such as

$$T = u(x^8 + ix^9) , \tag{3.19}$$

which makes a D7-brane localized at $x^8 = x^9 = 0$. When we place the D1-brane probe at $x^8 = x^9 = 0$, we obtain a massless Schwinger model as the effective theory on the D1-brane world-sheet. Actually, the D1-D7 system gives another realization of the Schwinger model. The open string stretched between the D1-brane and the D7-brane produces a Dirac fermion $\lambda = (\lambda_+, \lambda_-)^T$ charged under the U(1) gauge field on the D1-brane. When we consider n D1-branes on N coincident D7-branes, the massless fermions created by the 1-7 strings are as listed in table 3. While the low energy field contents on the D1-brane world-sheet are the same as those listed in table 1, the symmetry is different. Here only the subgroup

field	U(n)	SO(1,1)	SO(2)	U(N)
λ_+	**fund.**	1_+	1_+	**fund.**
λ_-	**fund.**	1_-	1_-	**fund.**

Table 3: The massless fields created by the 1-7 string. Here the U(n) and SO(1,1) are the gauge symmetry and the Lorentz symmetry on the D1-brane, respectively. The SO(2) corresponds to the rotation of the x^8-x^9 plane. The U(N) is the gauge symmetry on the D7-brane, which is seen as global symmetry of the D1-brane.

$\mathrm{SO}(2)\times\mathrm{U}(N)$ of the $\mathrm{U}(N)\times\mathrm{U}(N)$ chiral symmetry is manifest. The SO(2) symmetry, which is the rotational symmetry of the x^8-x^9 plane, is now interpreted as the axial $\mathrm{U}(1)_A$ symmetry as we can see in the table 3. This fact can also be seen from the tachyon configuration (3.19). The $\mathrm{U}(1)_A$ acts on the tachyon field as a phase transformation and it is translated to the rotation of the x^8-x^9 plane via the relation (3.19). Note that D7-brane charge is measured by the integration of dC_0 along the S^1 surrounding the D7-brane as

$$\int_{S^1} dC_0 = N\,. \tag{3.20}$$

This implies that C_0 is shifted as

$$C_0 \to C_0 + N\frac{\alpha}{\pi} \tag{3.21}$$

under the rotation $x^8+ix^9 \to e^{2i\alpha}(x^8+ix^9)$ which induces the $\mathrm{U}(1)_A$ transformation (3.3). This shift (3.21) precisely agrees with the shift (3.9) for $N=1$ case and hence the cancellation mechanism of the axial $\mathrm{U}(1)_A$ anomaly explained in section 3.2 also works in this case.

3.5 Adding flavors

It is not difficult to generalize our discussion to the N flavor case. The bosonized action for the N flavor (massive) Schwinger model is given as [23, 19]

$$S = \int d^2x \left\{ \frac{1}{2}\sum_{I=1}^{N}\partial_\mu\varphi_I\partial^\mu\varphi_I - \frac{m^2}{2}\left(\sum_{I=1}^{N}\varphi_I\right)^2 + cT\left(\sum_{I=1}^{N}\cos\left(2\sqrt{\pi}\,\varphi_I - \frac{\theta}{N}\right) - N\right)\right\}, \tag{3.22}$$

where $m^2 = g_{\mathrm{YM}}^2/\pi$ and we have assumed that the tachyon field is proportional to the unit matrix as $T^I{}_{\bar I} = T\,\delta^I{}_{\bar I}$ with $T\in\mathbb{R}_{\geq 0}$.

When the tachyon condenses as $T\to\infty$, each scalar field φ_I will take a discrete value

$$\varphi_I = \sqrt{\pi}\left(n_I + \frac{\theta}{2\pi N}\right), \qquad (n_I\in\mathbb{Z})\,, \tag{3.23}$$

just as our discussion in the one flavor case (3.13). Then, we obtain the energy density carried by this configuration as

$$\mathcal{E} = \frac{m^2}{2}\pi\left(\sum_{I=1}^{N} n_I + \frac{\theta}{2\pi}\right)^2 = \frac{g_s}{4\pi\alpha'}\left(\sum_{I=1}^{N} n_I + C_0\right)^2, \tag{3.24}$$

which is the same as the expression (3.14) with $n\equiv\sum_{I=1}^{N} n_I$. Therefore, this configuration again represents an $(n,1)$ string. Actually, the electric flux is related to the scalar fields as

$$F_{01} = -\frac{g_{\mathrm{YM}}^2}{\sqrt{\pi}}\sum_{I=1}^{N}\varphi_I = -g_{\mathrm{YM}}^2(n+C_0)\,, \tag{3.25}$$

which induces n units of fundamental string charge as expected.

4. QED$_2$ on a circle and the T-dual description

In this subsection we compactify the direction parallel to the D1-brane and give the string theory interpretation of some of the old results in the Schwinger model on S^1. We also consider applications of the field theory results to the D0-brane dynamics in the presence of a D8-$\overline{\text{D8}}$ pair by taking T-duality.

4.1 Schwinger model on a circle

Here we recapitulate the prescription given in [31]–[33] for the massless Schwinger model on a circle of radius R. Since the spatial coordinate x^1 is now periodic as $x^1 \sim x^1 + 2\pi R$, the spatial component of the gauge field A_1 also becomes a periodic variable with period $1/R$ via the gauge transformation

$$A_1 \sim A_1 + i\, e^{inx/R}(\partial_1 e^{-inx/R}) = A_1 + \frac{n}{R}\,. \tag{4.1}$$

Following [31, 32], we choose a gauge with

$$\partial_1 A_1 = 0\,. \tag{4.2}$$

Note that we cannot make $A_1 = 0$ by a gauge transformation, since $\exp\left(i \oint A_1 dx^1\right)$ is a gauge invariant quantity. The equation of motion for A_0 is now

$$\partial_1 F_{01} = -(\partial_1)^2 A_0 = g_{\rm YM}^2 \lambda^\dagger \lambda\,. \tag{4.3}$$

Then, standard manipulations lead to the hamiltonian

$$H = \int_0^{2\pi R} dx^1 \left\{ \frac{1}{2g_{\rm YM}^2}(\partial_0 A_1)^2 - \lambda^\dagger i\sigma^3(\partial_1 + iA_1)\lambda + \frac{g_{\rm YM}^2}{2}(\lambda^\dagger\lambda)\frac{1}{-\partial_1^2}(\lambda^\dagger\lambda) \right\}. \tag{4.4}$$

Here we have eliminated A_0 in the hamiltonian using (4.3).

It is also useful to work in the momentum representation. The Fourier expansion of fermion fields at a fixed time slice, say $x^0 = 0$, is given by

$$\lambda(x) = \frac{1}{\sqrt{2\pi R}} \sum_{k\in\mathbb{Z}} a_k\, e^{i\frac{k}{R}x^1}\,, \tag{4.5}$$

where $a_k = (a_{+,k}, a_{-,k})^T$ satisfy the canonical anti-commutation relations

$$\{a^\dagger_{\alpha,k}, a_{\beta,l}\} = \delta_{kl}\delta_{\alpha\beta}\,, \qquad \{a_{\alpha,k}, a_{\beta,l}\} = \{a^\dagger_{\alpha,k}, a^\dagger_{\beta,l}\} = 0\,. \tag{4.6}$$

Then, the hamiltonian (4.4) is written as

$$H = \frac{2\pi R}{2g_{\rm YM}^2}(\partial_0 A_1)^2 + \sum_{k\in\mathbb{Z}}\left(\frac{k}{R} + A_1\right) a_k^\dagger \sigma^3 a_k + \frac{g_{\rm YM}^2}{2}\sum_{k\neq 0} j^0(k)\frac{R^2}{k^2} j^0(-k)\,. \tag{4.7}$$

Note that $\partial_0 A_1$ is the zero-momentum component of the electric field F_{01} in our gauge (4.2). From (4.7), we see that the operators $a^\dagger_{\pm,k}$ and $a_{\pm,k}$ are creation and annihilation operators for a positive/negative chirality particle of momentum k/R and energy $\pm(k/R + A_1)$, respectively.

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The bosonization techniques can also be applied to this system. The relations (3.6) and (3.8) in the momentum representation are

$$\varphi(k) = \begin{cases} \dfrac{1}{\sqrt{2R}} \dfrac{2\pi R}{g_{\rm YM}^2}\, \partial_0 A_1 \;\; (k=0) \\ -i\sqrt{\pi}\dfrac{R}{k}\, j^0(k) \quad (k \neq 0) \end{cases} \tag{4.8}$$

$$\pi_\varphi(k) = \sqrt{\pi}\, j_5^0(k)\,. \tag{4.9}$$

Then, it is easy to see that the first and the third terms in the hamiltonian (4.7) correspond to the $k=0$ and $k \neq 0$ parts of the mass term of the scalar field φ, respectively, and we have

$$\frac{2\pi R}{2g_{\rm YM}^2}(\partial_0 A_1)^2 + \frac{g_{\rm YM}^2}{2}\sum_{k\neq 0} j^0(k)\frac{R^2}{k^2} j^0(-k) = \frac{g_{\rm YM}^2}{2\pi}\sum_{k\in\mathbb{Z}} \varphi(k)\varphi(-k)\,. \tag{4.10}$$

The second term in the hamiltonian (4.7) is more involved and one should carefully regularize the infinite sum. Here we just present the result shown in [31];

$$\sum_{k\in\mathbb{Z}} \left(\frac{k}{R} + A_1\right) a_k^\dagger \sigma^3 a_k = \frac{1}{2}\sum_{k\in\mathbb{Z}} \left(\pi_\varphi^\dagger(k)\pi_\varphi(k) + \left(\frac{k}{R}\right)^2 \varphi^\dagger(k)\varphi(k)\right), \tag{4.11}$$

neglecting an additive constant term. The equations (4.10) and (4.11) show that the hamiltonian (4.7) is equivalent to that for the free scalar field

$$H = \frac{1}{2}\sum_{k\in\mathbb{Z}} \left(\pi_\varphi^\dagger(k)\pi_\varphi(k) + \left(\frac{k}{R}\right)^2 \varphi^\dagger(k)\varphi(k) + \frac{g_{\rm YM}^2}{\pi}\varphi^\dagger(k)\varphi(k)\right). \tag{4.12}$$

4.2 Fermion Fock space and the Hanany-Witten effect

The fermion Fock space for the Schwinger model on S^1 is constructed by acting the operators $a_{\pm,k}^\dagger$ and $a_{\pm,k}$ satisfying (4.6) on a Fock vacuum defined by

$$\begin{aligned} a_{+,k}\,|\, M,N;A_1\,\rangle &= 0 \quad (\text{for } \;k > M)\,, \\ a_{+,k}^\dagger\,|\, M,N;A_1\,\rangle &= 0 \quad (\text{for } \;k \leq M)\,, \\ a_{-,k}\,|\, M,N;A_1\,\rangle &= 0 \quad (\text{for } \;k < N)\,, \\ a_{-,k}^\dagger\,|\, M,N;A_1\,\rangle &= 0 \quad (\text{for } \;k \geq N) \end{aligned} \tag{4.13}$$

for a fixed value of $A_1 \in [0, 1/R]$. By definition, these states satisfy

$$|\, M,N;A_1\,\rangle = a_{+,M}^\dagger\,|\, M-1,N;A_1\,\rangle = a_{-,N-1}\,|\, M,N-1;A_1\,\rangle\,. \tag{4.14}$$

In addition, since the gauge transformation with $g = e^{ix/R}$ induces $A_1 \to A_1 + 1/R$ and $a_{\pm,k} \to a_{\pm,k+1}$, the Fock vacua satisfy the boundary condition

$$|\, M,N;0\,\rangle = \left|\, M-1,N-1;\frac{1}{R}\right\rangle. \tag{4.15}$$

The total electric charge and axial charge is given by

$$Q = Q_+ + Q_- \,, \qquad Q_5 = Q_+ - Q_- \,, \tag{4.16}$$

respectively, where

$$Q_\pm \equiv \int_0^{2\pi R} dx^1 \, \lambda_\pm^\dagger(x)\lambda_\pm(x) = \sum_{k\in\mathbb{Z}} a_{\pm,k}^\dagger a_{\pm,k} \,. \tag{4.17}$$

With an appropriate regularization, it was shown in [31] that

$$Q_+ \,|\, M, N; A_1 \,\rangle \;=\; \left(M + RA_1 + \frac{1}{2}\right) |\, M, N; A_1 \,\rangle \tag{4.18}$$

$$Q_- \,|\, M, N; A_1 \,\rangle \;=\; \left(-N - RA_1 + \frac{1}{2}\right) |\, M, N; A_1 \,\rangle \,. \tag{4.19}$$

Since the equation (4.3) implies $Q = 0$, we take $N = M + 1$ and define a state

$$\left| \widetilde{A} \right\rangle = |\, M, M+1; A_1 \,\rangle \,, \tag{4.20}$$

where $\widetilde{A} = A_1 + \frac{M}{R}$. Note that the right hand side depends only on $\widetilde{A}$ because of the condition (4.15) and hence the state $\left| \widetilde{A} \right\rangle$ is well-defined. From (4.14), it satisfies

$$\left| \widetilde{A} + 1 \right\rangle = a_{+,M+1}^\dagger a_{-,M+1} \left| \widetilde{A} \right\rangle \,. \tag{4.21}$$

The axial charge of this state is given by

$$Q_5 \left| \widetilde{A} \right\rangle = 2\left(R\widetilde{A} + \frac{1}{2}\right) \left| \widetilde{A} \right\rangle \,. \tag{4.22}$$

Then, the hamiltonian for the wave function $\psi(\widetilde{A}) = \left\langle \widetilde{A} \middle| \psi \right\rangle$ becomes [31]

$$\left\langle \widetilde{A} \middle| H \middle| \psi \right\rangle = \left(-\frac{g_{\mathrm{YM}}^2}{4\pi R} \frac{\partial^2}{\partial \widetilde{A}^2} + V(\widetilde{A}) \right) \psi(\widetilde{A}) \,, \tag{4.23}$$

where

$$V(\widetilde{A}) = \frac{1}{R}\left(R\widetilde{A} + \frac{1}{2}\right)^2 \,. \tag{4.24}$$

Here, only the zero-momentum part of the hamiltonian (4.12) contributes and we have used the relations

$$\varphi(k=0) \;=\; \frac{1}{\sqrt{2R}} \frac{2\pi R}{g_{\mathrm{YM}}^2} \partial_0 A_1 = \frac{-i}{\sqrt{2R}} \frac{\partial}{\partial \widetilde{A}} \,, \tag{4.25}$$

$$\pi_\varphi(k=0) \;=\; \frac{1}{\sqrt{2R}} Q_5 = \sqrt{\frac{2}{R}} \left(R\widetilde{A} + \frac{1}{2}\right) \tag{4.26}$$

as the operators acting on $\left| \widetilde{A} \right\rangle$.

operator	energy	winding	charge	string
$a^\dagger_{+,k}$	$T_1(2\pi k\widehat{R}+X^1)$	$\left(k+X^1/2\pi\hat{R}\right)$	$+$	$0-8$ string
$a_{+,k}$	$-T_1(2\pi k\widehat{R}+X^1)$	$-\left(k+X^1/2\pi\hat{R}\right)$	$-$	$8-0$ string
$a^\dagger_{-,k}$	$-T_1(2\pi k\widehat{R}+X^1)$	$\left(k+X^1/2\pi\hat{R}\right)$	$+$	$0-\overline{8}$ string
$a_{-,k}$	$T_1(2\pi k\widehat{R}+X^1)$	$-\left(k+X^1/2\pi\hat{R}\right)$	$-$	$\overline{8}-0$ string

Table 4: The fermionic operators in (4.6) carry the energy, winding number, and charge as in this table. We can identify the states created by these operators as the states generated by the corresponding oriented strings stretched between D0 and D8 or D0 and $\overline{\text{D8}}$ as in this table.

It would be more convenient to express these in the T-dual picture. When we T-dualize the compactified x^1-direction, we obtain a system with a D0-brane and a D8-$\overline{\text{D8}}$ pair, which are localized on a circle of radius $\widehat{R}\equiv\alpha'/R$. The gauge field A_1 on the D1-brane is related to the position X^1 of the D0-brane as $A^1=\frac{X^1}{2\pi\alpha'}$ [13]. (See also section 5.1) The gauge transformation (4.1) implies the periodicity of the T-dualized circle $X^1\sim X^1+2\pi\widehat{R}$. We also introduce the notation $\widetilde{X}\equiv 2\pi\alpha'\widetilde{A}$ which corresponds to the position of the D0-brane in the covering space of the S^1. Then the potential energy (4.24) of the system is rewritten as

$$V(\widetilde{X})=\frac{T_1}{2\pi\widehat{R}}\left(\widetilde{X}+\pi\widehat{R}\right)^2 . \tag{4.27}$$

The momentum index k carried by the operators $a_{\pm,k}$ and $a^\dagger_{\pm,k}$ is mapped to the winding number and the energy $\pm(k/R+A_1)=\pm T_1(2\pi k\widehat{R}+X^1)$ is interpreted as the length times the tension $T_1\equiv 1/2\pi\alpha'$ of the 0-8 or 0-$\overline{8}$ strings.

In the classical picture, the excited energy of the system with a static D0-brane is expected to be proportional to the length of the strings attached on it. Since the minimum of the potential (4.27) is given by $\widetilde{X}=-\pi\widehat{R}$, it is natural to interpret that the state $\left|\widetilde{X}\right\rangle\equiv\left|\widetilde{A}\right\rangle$ with $-2\pi\widehat{R}<\widetilde{X}<0$ corresponds to a configuration without any strings attached on the D0-brane. The first excited states are

$$a^\dagger_{+,0}\left|\widetilde{X}\right\rangle , \qquad a_{+,-1}\left|\widetilde{X}\right\rangle , \qquad a^\dagger_{-,-1}\left|\widetilde{X}\right\rangle , \qquad a_{-,0}\left|\widetilde{X}\right\rangle , \tag{4.28}$$

which correspond to the configurations with one string attached on the D0-brane as depicted in figure 1. Note however that these states are not physical since the total electric charge does not vanish.

What happens when the D0-brane turns around the circle? As we have seen in (4.21), $2\pi\widehat{R}$ shift of the position of the D0-brane can be expressed as

$$\left|\widetilde{X}+2\pi\widehat{R}\right\rangle=a^\dagger_{+,0}a_{-,0}\left|\widetilde{X}\right\rangle \tag{4.29}$$

for $-2\pi\widehat{R}<\widetilde{X}<0$. The right hand side of this equation corresponds to the configuration with a 0-8 string and a $\overline{8}$-0 string attached on the D0-brane. This relation can be understood as a result of the Hanany-Witten effect [34].[5] It suggests that when the D0-brane crosses

[5] See also [35]–[42] for the discussions in closely related situations.

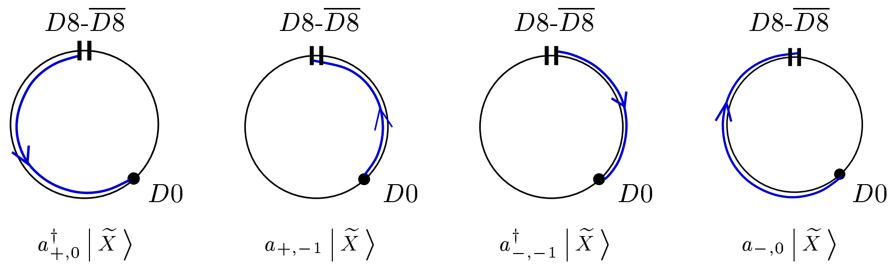

Figure 1: From table 4, we can identify the fermionic states created by the operators in (4.6) as the states generated by the corresponding strings as in this figure.

the D8-brane or $\overline{\text{D8}}$-brane, a fundamental string is created between the D0-brane and the D8-brane or $\overline{\text{D8}}$-brane. Actually, the axial charge is shifted by two when the D0-brane travels around the circle due to the axial $\mathrm{U}(1)_A$ anomaly as we can explicitly see from (4.22) which implies the creation of a pair of fundamental strings (a 0-8 string and a $\overline{8}$-0 string) as discussed in [35]. It is also interesting to note that we cannot make a configuration with two or more strings of the same type attached on the D0-brane because of the Pauli's exclusive principle, since the operators $a_{\pm,k}$ are fermionic. This observation is used as the explanation of the s-rule in [41, 43], though our configuration is non-supersymmetric.

When the D0-brane is turned k times around the circle, the system will gain the energy of

$$V(2\pi k\widehat{R}+\widetilde{X})-V(\widetilde{X}) = 2T_1 k\left(\widetilde{X}+(k+1)\pi\widehat{R}\right) \tag{4.30}$$

$$= 2T_1\sum_{l=1}^{k}\left(2\pi l\widehat{R}+\widetilde{X}\right). \tag{4.31}$$

Note that the length of the strings created by the Hanany-Witten effect during the process is $(2\pi l\widehat{R}+\widetilde{X})$ $(l=1,2,\ldots,k)$. The expression (4.31) clearly shows that it is equal to the energy carried by these strings.

In order to obtain the precise form of the potential (4.27), we have to take into account the quantum effects. A string theoretical derivation of the potential will be given in the next subsection.

4.3 The D0-brane potential from stringy calculations

Let us suppose $\widetilde{X} = 2\pi k\widehat{R}+X$ with $k\in\mathbb{Z}$, $-2\pi\widehat{R}<X<0$ and extract the quantum correction of the potential

$$\Delta V(\widetilde{X}) \equiv V(\widetilde{X})-2T_1\sum_{l=1}^{k}\left(2\pi l\widehat{R}+X\right) = \frac{T_1}{2\pi\widehat{R}}\left(X+\pi\widehat{R}\right)^2 \tag{4.32}$$

$$= \frac{2\widehat{R}T_1}{\pi}\sum_{n=1}^{\infty}\frac{1}{n^2}\cos\left(\frac{n\widetilde{X}}{\widehat{R}}\right)+\text{const.}, \tag{4.33}$$

$$= \frac{1}{\pi^2 R}\sum_{n=1}^{\infty}\frac{1}{n^2}\cos\left(2\pi nR\widetilde{A}\right)+\text{const.}\,, \tag{4.34}$$

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by subtracting the energy carried by the fundamental strings (4.31) from the potential $V(\widetilde{X})$ (4.27). The expression (4.34) is obtained in [32] from the one-loop diagrams of fermions in the Schwinger model on S^1. It would be instructive to rederive the D0-brane potential (4.33) in terms of string theory.

The potential of the D0-brane in the presence of the D8-$\overline{\text{D8}}$ pair is produced by the exchange of closed strings represented by the amplitude of the cylinder diagram

$$\mathcal{A}(\widetilde{X}) = \int_0^\infty d\tau \left(\langle\, \mathrm{D0}\,|\, e^{-\pi\tau(L_0+\widetilde{L_0})}\,|\,\mathrm{D8}\,\rangle + \langle\, \mathrm{D0}\,|\, e^{-\pi\tau(L_0+\widetilde{L_0})}\,\big|\,\overline{\mathrm{D8}}\,\rangle \right), \tag{4.35}$$

where $|\,\mathrm{D0}\,\rangle$, $|\,\mathrm{D8}\,\rangle$ and $\big|\,\overline{\mathrm{D8}}\,\big\rangle$ are the boundary states corresponding to the D0-brane, D8-brane and $\overline{\text{D8}}$-brane, respectively. This amplitude for the non-compact case has been calculated in [37] and the result is

$$\mathcal{A}_{\text{non-cpt}}(X) = -\frac{V_0}{(8\pi^2\alpha')^{1/2}} \int_0^\infty d\tau\, \tau^{-1/2} e^{-X^2/2\pi\alpha'\tau}, \tag{4.36}$$

where X is the distance between the D0-brane and the D8-$\overline{\text{D8}}$ pair and V_0 is the volume factor. Note that we haven't taken $\alpha \to 0$ limit here. The contribution of the massive closed strings automatically cancels in this cylinder amplitude.

When we compactify the direction transverse to the branes as in the previous subsection, we have to take into account the contributions from the infinite copies of the D8-$\overline{\text{D8}}$ pairs in the covering space as

$$\mathcal{A}(\widetilde{X}) = \sum_{n\in\mathbb{Z}} \mathcal{A}_{\text{non-cpt}}\left(\widetilde{X} + 2\pi\widehat{R}n\right). \tag{4.37}$$

Then the potential will become

$$\begin{aligned}
\Delta V(\widetilde{X}) &= -\mathcal{A}(\widetilde{X})/V_0 && (4.38)\\
&= \frac{1}{(8\pi\alpha')^{1/2}} \sum_{n\in\mathbb{Z}} \int_0^\infty d\tau\, \tau^{-1/2} e^{-(\tilde{X}+2\pi\hat{R}n)^2/2\pi\alpha'\tau} && (4.39)\\
&= \frac{1}{(8\pi\alpha')^{1/2}} \frac{\sqrt{2\alpha'}}{2\widehat{R}} \sum_{n\in\mathbb{Z}} \int_0^\infty d\tau\, e^{\frac{\pi\alpha' n^2}{2\hat{R}^2}\tau - \frac{n\tilde{X}}{\hat{R}} i} && (4.40)\\
&= \frac{2\widehat{R}}{\pi(2\pi\alpha')} \sum_{n=1}^\infty \frac{1}{n^2} \cos\left(\frac{n\widetilde{X}}{\widehat{R}}\right) + \text{const.}, && (4.41)
\end{aligned}$$

reproducing (4.33) as promised. Here we have used the Poisson resummation formula in the equality between (4.39) and (4.40).

4.4 Quantum mechanics of the D0-brane in the D8-$\overline{\text{D8}}$ background

Here we continue our discussion in the T-dual picture. The wave function $\psi(\widetilde{X}) = \left\langle \widetilde{X} \,\middle|\, \psi \right\rangle$ is now regarded as the wave function of the D0-brane. T-dualizing the hamiltonian (4.23), we obtain the Schrödinger equation for the D0-brane wave function as

$$i\frac{\partial}{\partial t}\psi(\widetilde{X}) = \left(-\frac{1}{2T_0}\frac{\partial^2}{\partial \widetilde{X}^2} + V(\widetilde{X}) \right) \psi(\widetilde{X}), \tag{4.42}$$

where $T_0 \equiv 1/\widehat{g}_s l_s$ $(\widehat{g}_s \equiv g_s l_s/R)$ is the mass of the D0-brane as $\widehat{g}_s$ is the string coupling in the T-dual picture. Since the potential (4.27) is quadratic with respect to $\widetilde{X} + \pi\widehat{R}$, this Schrödinger equation is the same as that for a harmonic oscillator. The ground state is given by

$$\psi(\widetilde{X}) = \mathcal{N} \exp\left\{ -\frac{(\widetilde{X} + \pi\widehat{R})^2}{\sqrt{8\pi^2 \widehat{g}_s l_s^3 \widehat{R}}} \right\} \tag{4.43}$$

$$= \mathcal{N} \exp\left\{ -\frac{\sqrt{\pi}R}{g_{\mathrm{YM}}} \left(\widetilde{A} + \frac{1}{2R} \right)^2 \right\}, \tag{4.44}$$

where $\mathcal{N}$ is a normalization constant.

According to (4.43), the D0-brane is trapped around $\widetilde{X} = -\pi\widehat{R}$ with a width of order $(\widehat{g}_s l_s^3 \widehat{R})^{1/4} \sim \alpha'(g_{\mathrm{YM}}/R)^{1/2}$. Though this width of the quantum fluctuation will formally become zero if we take the decoupling limit $\alpha' \to 0$ keeping g_{YM} and R fixed, the ratio to the radius $\widehat{R} = \alpha'/R$ remains finite as $\delta\widetilde{X}/\widehat{R} \sim (R g_{\mathrm{YM}})^{1/2}$.

5. T-duality, matrix and lattice regularization

One of the advantages of the realization of a field theory in string theory is that we can apply various dualities known in string theory. As an application of our realization of QED on a D-brane world-volume, we can apply T-duality and obtain its matrix theory description. Actually, this method can be applied to a wide variety of field theories including the realistic QCD, even though we haven't obtained its string theory realization.[6] We also show that when we regularize the theory by replacing infinite size matrices to finite ones, we naturally obtain usual lattice gauge theory.

Although we use some terminology appeared before to make connections with the previous sections, most part of this section can be read independently. Actually, this section is intended to be readable for the readers who may not be familiar with string theory.

Throughout this section we have set $2\pi\alpha' = 1$.

5.1 T-duality and matrix theory description of QED

Let us first consider 4 dimensional QED with N_f flavors as an exercise, whose action is

$$S_{\mathrm{D3}} = \int d^4x \left(-\frac{1}{4g_{\mathrm{YM}}^2} F_{\mu\nu}F^{\mu\nu} + i\overline{\lambda}_I \gamma^\mu D_\mu \lambda^I \right), \tag{5.1}$$

where $\mathrm{D}_\mu = \partial_\mu + iA_\mu$. As discussed in section 2, it is realized as the world-volume theory of a D3-brane in the background of N_f D9-$\overline{\mathrm{D9}}$ pairs. When we compactify one of the spatial directions, say x^3, to S^1 of radius R and take T-duality along it, we obtain a system with a D2-brane and N_f D8-$\overline{\mathrm{D8}}$ pairs localized on the S^1 of radius $\widehat{R} = 1/2\pi R$ and the 4 dimensional QED is mapped to a 3 dimensional theory realized on the D2-brane.

[6]See [44, 45] for recent progress.

Then, we can apply the prescription given in [9]–[12] to obtain this map. Let us explain this prescription shortly. The 3 dimensional world-volume theory consists of the following infinite size matrix valued fields

$$(A_\alpha(x^\beta))^A{}_B\,, \qquad (X^3(x^\beta))^A{}_B\,, \qquad (\lambda^I(x^\beta))^A{}_B\,, \qquad (\alpha,\beta=0,1,2;\ A,B\in\mathbb{Z})\,, \tag{5.2}$$

satisfying the constraints

$$\begin{aligned}
(A_\alpha(x^\beta))^{A+1}{}_{B+1} &= (A_\alpha(x^\beta))^A{}_B\,, && (5.3)\\
(X^3(x^\beta))^{A+1}{}_{B+1} &= (X^3(x^\beta))^A{}_B + 2\pi\widehat{R}\,\delta^A{}_B\,, && (5.4)\\
(\lambda^I(x^\beta))^{A+1}{}_{B+1} &= (\lambda^I(x^\beta))^A{}_B\,. && (5.5)
\end{aligned}$$

A trivial solution of the constraint (5.4) is given by a diagonal matrix $(\Delta^3)^A{}_B = 2\pi\widehat{R}A\,\delta^A{}_B$. Using this matrix, the constraint (5.4) can also be written as

$$(\widehat{X}^3(x^\beta))^{A+1}{}_{B+1} = (\widehat{X}^3(x^\beta))^A{}_B\,, \qquad \widehat{X}^3(x^\beta) \equiv X^3(x^\beta) - \Delta^3\,. \tag{5.6}$$

From these constraints, we can choose a row (e.g. $B=0$) of each matrix in (5.2) as the independent degrees of freedom.

The explicit correspondence between the 4 dimensional fields and the 3 dimensional fields is given by the Fourier transformation as

$$\begin{aligned}
A_\alpha(x^\beta,x^3) &= \sum_{A\in\mathbb{Z}} (A_\alpha(x^\beta))^A{}_0\, e^{iAx^3/R}\,, && (5.7)\\
A_3(x^\beta,x^3) &= \sum_{A\in\mathbb{Z}} (X_3(x^\beta))^A{}_0\, e^{iAx^3/R}\,, && (5.8)\\
\lambda^I(x^\beta,x^3) &= \sum_{A\in\mathbb{Z}} (\lambda^I(x^\beta))^A{}_0\, e^{iAx^3/R}\,. && (5.9)
\end{aligned}$$

Note that the gauge choice $\partial_3 A_3 = 0$ we used in (4.2) for the Schwinger model implies X^3 is diagonal and $A_3 = (X^3)^0{}_0$ which is the relation we used in section 5.1. The action for the 3 dimensional description is

$$\begin{aligned}
S_{\mathrm{D2}} = \frac{2\pi R}{\mathrm{Tr}\,1}\int d^3x\ \mathrm{Tr}\bigg(&-\frac{1}{4g_{\mathrm{YM}}^2}F_{\alpha\beta}F^{\alpha\beta} + i\overline{\lambda}_I\gamma^\alpha D_\alpha\lambda^I - \\
&-\frac{1}{4g_{\mathrm{YM}}^2}[D_\alpha, X_3][D^\alpha, X^3] + i\overline{\lambda}_I\gamma^3\widehat{D}_3\lambda^I\bigg)\,,
\end{aligned} \tag{5.10}$$

where $\widehat{D}_3\lambda^I \equiv i([\Delta^3,\lambda^I] + \widehat{X}^3\lambda^I)$. Here we need to divide the trace by $\mathrm{Tr}\,1$ to extract one component out of the infinite copies in the covering space. One can easily show that the actions (5.1) and (5.10) are equal under the correspondence (5.7)$\sim$ (5.9).

We can T-dualize the rest of the directions in the same way. When we take T-duality along all the space-time directions,[7] we obtain a matrix theory description of the 4 dimensional QED based on the D(−1)-branes in the presence of D5-$\overline{\text{D5}}$ pairs. Then, the

[7]To perform T-duality along the time direction, we consider a Wick rotated theory and formally apply the T-duality rules to the euclidean time direction.

world-volume is 0 dimensional and the theory is described by infinite size matrices $(X^\mu)^A{}_B$ ($\mu = 0, \ldots, 3$) and $(\lambda^I)^A{}_B$ constrained as

$$(X^\mu)^{A+\hat{\nu}}{}_{B+\hat{\nu}} = (X^\mu)^A{}_B + 2\pi\widehat{R}\,\delta^\mu{}_\nu \delta^A{}_B\,, \tag{5.11}$$
$$(\lambda^I)^{A+\hat{\nu}}{}_{B+\hat{\nu}} = (\lambda^I)^A{}_B\,. \tag{5.12}$$

Here the indices A, B are labeled by $\mathbb{Z}^4$ as $A = (a_0, a_1, a_2, a_3)$ with $a_0, a_1, a_2, a_3 \in \mathbb{Z}$ and we have used the notation $\widehat{\nu} = (\delta_{\nu 0}, \delta_{\nu 1}, \delta_{\nu 2}, \delta_{\nu 3})$.

The action is now

$$S_{\mathrm{D}(-1)} = \frac{(2\pi R)^4}{\mathrm{Tr}\,1}\,\mathrm{Tr}\left(\frac{1}{4g_{\mathrm{YM}}^2}[X_\mu, X_\nu][X^\mu, X^\nu] + i\overline{\lambda}_I\gamma^\mu\widehat{D}_\mu\lambda^I\right), \tag{5.13}$$

where the covariant derivative of λ^I is given as

$$\widehat{D}_\mu\lambda^I \equiv i([\Delta_\mu, \lambda^I] + \widehat{X}_\mu\lambda^I) = i(X_\mu\lambda^I - \lambda^I\Delta_\mu)\,, \tag{5.14}$$
$$(\Delta_\mu)^A{}_B \equiv 2\pi\widehat{R}\,a_\mu\delta^A{}_B\,. \tag{5.15}$$

We have explained the T-duality prescription using 4 dimensional QED. The generalization to arbitrary gauge theory is straightforward since it is just a Fourier transformation.

5.2 Matrix regularization and lattice gauge theory

The matrix theory description of the gauge field theory (5.13) is not practical for computer simulations, since the size of matrices is infinite. Here we consider a natural regularization to cut off the size of the matrices.

We again use the 4 dimensional QED considered in the previous subsection as an illustrative example, though the generalization to other field theory is straightforward. The basic idea is to replace the group $\mathbb{Z}^4$ which labels the matrix indices with a finite group $\mathbb{Z}_N^4$. Then, the problem is that the constraint (5.11) does not make sense if the indices A, B were labeled by $\mathbb{Z}_N^4$. Actually, it implies

$$(X^\mu)^{A+N\hat{\mu}}{}_{B+N\hat{\mu}} = (X^\mu)^A{}_B + 2\pi\widehat{R}N\,\delta^A{}_B\,. \tag{5.16}$$

While the left hand side should be equal to $(X^\mu)^A{}_B$ when the indices A, B are $\mathbb{Z}_N^4$ valued, the second term of the right hand side $2\pi\widehat{R}N$ cannot be zero. A possible modification is to exponentiate the matrices X^μ and introduce $\mathrm{U}(N)$ matrices

$$U_\mu \equiv e^{iaX^\mu}\,, \qquad a \equiv \frac{1}{\widehat{R}N} \tag{5.17}$$

which are subject to the constraints

$$\begin{aligned}(U_\mu)^{A+\hat{\mu}}{}_{B+\hat{\mu}} &= \omega\,(U_\mu)^A{}_B\,,\\ (U_\mu)^{A+\hat{\nu}}{}_{B+\hat{\nu}} &= (U_\mu)^A{}_B\,, \quad \text{for} \quad \mu \neq \nu\,,\end{aligned} \tag{5.18}$$

where $\omega = e^{2\pi i/N}$. Here $a = 1/\widehat{R}N$ is a small parameter which becomes zero in the large N limit. We will soon see that this parameter a is interpreted as the lattice spacing. Note that $aN = 2\pi R$ is the length of the space of the 4 dimensional theory, which is fixed in the large N limit.

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We can construct a (naive) action which reproduce (5.13) at leading order in a as

$$S = a^4 \left(\frac{1}{4g_{\mathrm{YM}}^2 a^4} \sum_{\mu \neq \nu} \mathrm{Tr} \left(U_{\mu\nu} + U_{\mu\nu}^\dagger - 2 \right) + \frac{1}{2ia} \mathrm{Tr} \left(\overline{\lambda}_I \gamma^\mu (U_\mu \lambda^I \Omega_\mu^{-1} - U_\mu^\dagger \lambda^I \Omega_\mu) \right) \right), \tag{5.19}$$

where $U_{\mu\nu} = U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger$ and Ω_μ is defined as

$$(\Omega_\mu)^A{}_B = (e^{ia\Delta_\mu})^A{}_B = \omega^{a_\mu} \delta^A{}_B\,. \tag{5.20}$$

This action is closely related to the unitary matrix model proposed in [46, 47, 48]. Just like the usual lattice theory, this action (5.19) suffers from the fermion doubling problem. It could be avoided in a similar way as in the usual lattice theory, say adding a Wilson term, though we will not discuss it here. This action also reminds us of the Eguchi-Kawai model [49], since the action (5.19) looks like that obtained as the dimensional reduction of 4 dimensional U(N) gauge theory. But, here we have the constraints (5.12) and (5.18) for the matrices.

Now we claim that the action (5.19) together with the constraints (5.12) and (5.18) defines the usual lattice regularization of the action (5.1). Actually, the matrix description (5.19) is the momentum representation of the lattice theory. In order to make a Fourier transformation to get the coordinate space representation, it is useful to introduce the following bracket notation:

$$\langle X|A\rangle = \frac{1}{N^2}\omega^{A\cdot X} \equiv \frac{1}{N^2}\omega^{a_0 x^0}\omega^{a_1 x^1}\omega^{a_2 x^2}\omega^{a_3 x^3} \tag{5.21}$$

$$\sum_{X\in\mathbb{Z}_N^4} \langle A|X\rangle\langle X|B\rangle = \delta_{AB}, \qquad \sum_{A\in\mathbb{Z}_N^4} \langle X|A\rangle\langle A|Y\rangle = \delta_{XY}\,, \tag{5.22}$$

where $X = (x^0, x^1, x^2, x^3) \in \mathbb{Z}_N^4$ is the coordinate space. The matrix element $(U_\mu)^A{}_B$ is written as $\langle A|U_\mu|B\rangle$ and its coordinate representation is

$$\langle X|U_\mu|Y\rangle = \sum_{A,B} \langle X|A\rangle\, \langle A|\, U_\mu \,|B\rangle\, \langle B|Y\rangle \tag{5.23}$$

$$= \frac{1}{N^4}\sum_{A,B} \omega^{A\cdot X - B\cdot Y} (U_\mu)^A{}_B\,. \tag{5.24}$$

Using the constraint (5.18), we have $(U_\mu)^A{}_B = \omega^{b_\mu}(U_\mu)^{A-B}{}_0$ and (5.24) can be calculated as

$$\langle X|\, U_\mu \,|Y\rangle = \frac{1}{N^4}\sum_{A,B} \omega^{A\cdot X - B\cdot(Y-\hat{\mu})} (U_\mu)^{A-B}{}_0 \tag{5.25}$$

$$= \frac{1}{N^4}\sum_{A'} (U_\mu)^{A'}{}_0\; \omega^{A'\cdot X} \sum_B \omega^{B\cdot(X+\hat{\mu}-Y)} \tag{5.26}$$

$$= U_\mu(X)\, \delta_{X+\hat{\mu},Y}\,, \tag{5.27}$$

where we have defined $U_\mu(X) \equiv \sum_A (U_\mu)^A{}_0\, \omega^{A\cdot X}$ and used the relation $\frac{1}{N^4}\sum_B \omega^{B\cdot(X+\hat{\mu}-Y)}$ $= \delta_{X+\hat{\mu},Y}$. Similarly, we obtain $\langle X|\, U_\mu^\dagger \,|Y\rangle = U_\mu^\dagger(Y)\delta_{X-\hat{\mu},Y}$ for the hermitean conjugate of the matrix U_μ.

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Then the trace of $U_{\mu\nu}$ can be calculated as

$$
\begin{aligned}
\mathrm{Tr}(U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger) &= \\
&= \sum_{X_1,X_2,X_3,X_4} \langle X_1|U_\mu|X_2\rangle\langle X_2|U_\nu|X_3\rangle\langle X_3|U_\mu^\dagger|X_4\rangle\langle X_4|U_\nu^\dagger|X_1\rangle \\
&= \sum_{X_1,X_2,X_3,X_4} U_\mu(X_1)\delta_{X_1+\hat{\mu},X_2}U_\nu(X_2)\delta_{X_2+\hat{\nu},X_3}U_\mu^\dagger(X_4)\delta_{X_3-\hat{\mu},X_4}U_\nu^\dagger(X_1)\delta_{X_4-\hat{\nu},X_1} \\
&= \sum_X U_\mu(X)\, U_\nu(X+\widehat{\mu})\, U_\mu^\dagger(X+\widehat{\nu})\, U_\nu^\dagger(X)\,, \qquad (5.28)
\end{aligned}
$$

which is the sum of the plaquettes over the space-time, and hence the first trace in the (5.19) gives the usual action for the gauge field on the lattice.

The matter part is also performed similarly. The matrix element of the fermion λ^I in the coordinate space is given by

$$
\langle X|\,\lambda^I\,|Y\rangle = \lambda^I(X)\,\delta_{X,Y}\,, \qquad (5.29)
$$

where we have used the constraint (5.12) and defined $\lambda^I(X) = \sum_A (\lambda^I)^A{}_0\,\omega^{A\cdot X}$. The matrix Ω_μ defined in (5.20) acts as a shift matrix

$$
\langle X|\,\Omega_\mu\,|Y\rangle = \delta_{X+\hat{\mu},Y}\,, \qquad (5.30)
$$

in the coordinate space. Then, the matter part of the action (5.19) becomes

$$
\mathrm{Tr}\left(\overline{\lambda}_I\gamma^\mu(U_\mu\lambda^I\Omega^{-1} - U_\mu^\dagger\lambda^I\Omega)\right) = \sum_X \overline{\lambda}_I(X)\gamma^\mu\left(U_\mu(X)\lambda^I(X+\widehat{\mu}) - U_\mu^\dagger(X)\lambda^I(X-\widehat{\mu})\right), \qquad (5.31)
$$

which is the standard (naive) fermion action for the lattice gauge theory.

A few comments are in order. A similar approach to the matrix description of the lattice gauge theory can be found in [50]–[52]. In these papers, the matrix variables are treated in the coordinate space ($\langle X|\,U_\mu\,|Y\rangle$ etc. in our notation). The essential step in our discussion is the finite size regularization of the infinite size matrices. We have replaced the $\mathbb{Z}$ valued matrix indices to $\mathbb{Z}_N$ valued ones. This is essentially the same procedure in the dimensional deconstruction [53], in which the regularization is obtained by replacing a cylinder ($\mathbb{R}\times\mathbb{R}/\mathbb{Z}$) geometry with an orbifold ($\mathbb{C}/\mathbb{Z}_N$)[54].

6. Discussion

We have seen that two-dimensional and four-dimensional QED are realized in string theory as the world-volume theory on a probe D1-brane and D3-brane, respectively, in the presence of D9-$\overline{\text{D9}}$ pairs in type IIB string theory. In particular, the field theoretical methods developed in QED_2 have been efficiently used to study some properties of the D-branes in this system.

Though we have concentrated on the D9-$\overline{\text{D9}}$ system and its T-dual cousins, it should be possible to study other unstable D-brane systems in a similar way. It would be interesting to make more general investigation on the unstable D-brane systems using the D-brane probes.

Our motivation to use a D1-brane as a probe is not only that the world-sheet theory is a solvable field theory, but also that it is mapped to a fundamental string under the S-duality in type IIB string theory. If we wish to study the D9-$\overline{\text{D9}}$ system in the strong coupling region, it is natural to take the D1-brane as a fundamental object. This is quite analogous to the S-duality between type I string theory and Heterotic SO(32) string theory, in which the field contents on the D1-brane in type I string theory is the same as those on the fundamental string in the Heterotic string theory [55]. The vertex operators for the ten dimensional gauge fields which can be read from (2.8) are the analog of that for the SO(32) gauge field in the fermionic construction of the Heterotic string. However, unlike the type I D1-brane, the field contents listed in table 1 do not seem to lead to a conformally invariant theory in a simple way even in the $n = 1$ case. It may be too naive to make such observation, since we have taken the weak coupling limit $g_s \to 0$ to determine the action which may not give a good description in the strong coupling region. We hope our probe analysis will be helpful to gain some insights to the strong coupling analysis of the unstable D-brane system in the future study.

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